



Formulating Problems for the Grades 5 & 6 in the Mathematik-Olympiade

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Situation at the Beginning of the 5th Grade

- The Primary Schools do not provide the young students with mathematical facts, methods, nor robust knowledge.
- Of course, an aptitude for mathematic reasoning can also not be expected.
- Therefore, interested kids cannot rely on more than their interest and, hopefully, on their curiosity and their intuition.
- Also, many of the more gifted kids are not accustomed to work on a problem beyond a easily acquired first idea – endurance is a capability to develop.

Consequences for the Formulation of Problems

- Keep the kids' curiosity alive, train their endurance level
- Teach mathematical reasoning
- Introduce new and extendable contents and open new fields of questions

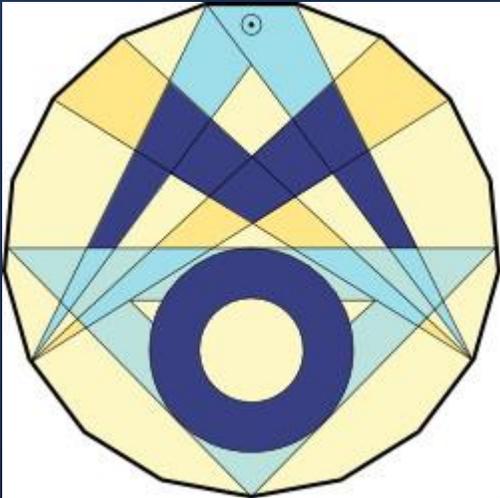


Fields of possible problems

- Number theory – prime numbers, factorization; equations
- Geometry – intersections of geometric objects, „counting geometry“, i.e. the construction of larger structures from smaller geometrical objects
- Combinatorics – permutations and combinations
- Logical problems – riddles, introduction of logical functions
- Algorithms – iterative procedures
- Methods – e.g. reverse calculation, systematic (tabellaric) work on equations



550511 – A First Encounter with an Algorithm (5th Grade, 1th Round)



An algorithm for generating sequences of numbers has the following rules:

o) Choose a number and write it on paper.

i) Add 5 to this number.

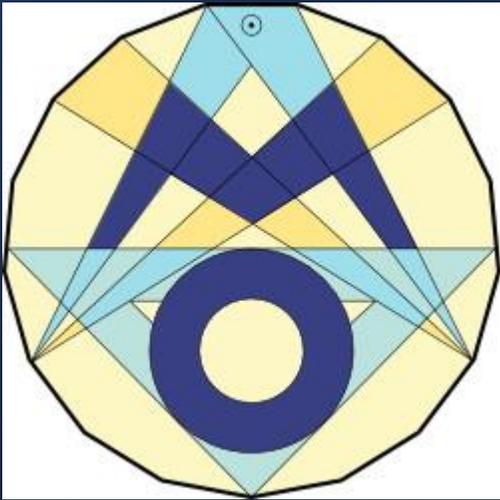
ii) Reduplicate the result of i).

iii) Subtract 10 from the result of ii)

iv) Write down the result and go to i).

- a) Start with the numbers 2, 6, and 7, consecutively. Repeat the sequence of the rules five times so that you get a sequence of six numbers for each of the starting numbers.
- b) Choose another two starting numbers, repeat the calculations and observe the number sequences generated. What can you find out? Justify your assumptions.
- c) Is there a starting number that leads to the number 104 after four repetitions of the algorithm? Give a justification.

600612 – Sums of Primes (6th Grade, 1st Round)

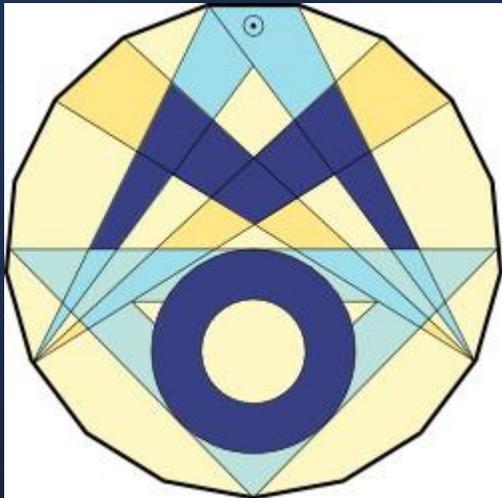


Since this is the 60th Mathematics Olympiad, let's have a look on the number 60:

This problem deals with the possibilities to sum up **different primes** to receive the sum 60.

- a) Find two **different primes** so that their sum is 60.
- b) Find three **different primes** so that their sum is 60.
- c) Find four **different primes** so that their sum is 60.
- d) Find five **different primes** so that their sum is 60.
- e) Inquire whether 60 can be written as a sum of six **different primes**.

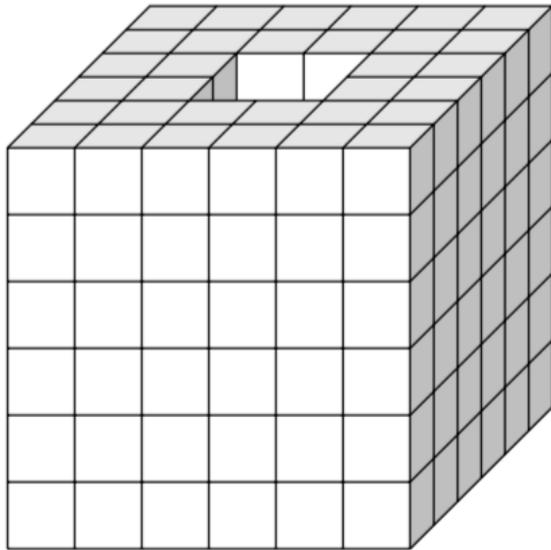
580533 –
Numbers and
Mirror
Numbers
(5th Grade,
3rd Round)



Given a number z , its mirror number z' consists of the same digits in reverse order. In this problem, the last digit of z shall not be zero.

- Give to $z = 952$ its mirror number z' and calculate the sum and the difference of z and z' .
- Another pair of z and z' has a sum of 88 and a difference of 36. Find z and mark z , the sum and the difference on a number line.
- Of a third pair of numbers z and z' its sum is 11121 and its difference is 7083. Find z and z' .

570534 – An Example of „Geometry by Counting“



A large cube is assembled of smaller cubes of equal size. Then some of the smaller cubes are removed.

The result is a structure we will call „BUILDING“ (see illustration on the left) The BUILDING looks the same viewed from top and from bottom; the removed smaller cubes form a cuboid. Therefore, you can look through the BUILDING in a vertical view.

a) How many of the small cubes form the BUILDING?

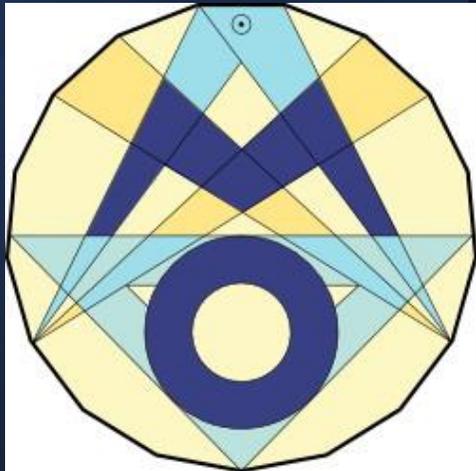
Now the BUILDING is immersed in a colour bath.

b) How many of the small cubes will show exactly one painted face?

c) How many of the small cubes have no coloured faces at all?

Two Additions:

580611 – A Logical Riddle



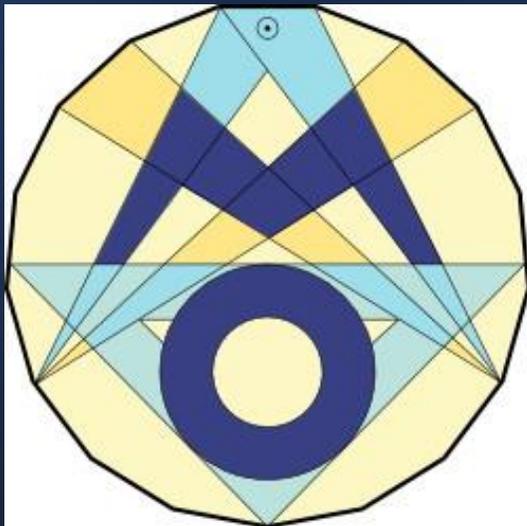
During a cruise, seven ladies meet on the sundeck to sip their tea. Since the cruise has already lasted for a long time, they wonder about the actual day of the week. They make the following statements:

- Agnes: The day after tomorrow will be Wednesday.
- Bertha: No. Wednesday is today.
- Clara: You're both wrong. Tomorrow it will be Wednesday.
- Doris: Nonsense. Today we do not have Monday, not Tuesday, nor Wednesday.
- Elise: I'm rather sure: Yesterday was Thursday.
- Frieda: No, no, no. Yesterday was Tuesday.
- Gertrud: All that I know is – yesterday was not Saturday.

Unfortunately, six of the ladies are wrong, only one statement is true. On which day of the week the conversation takes place?

Two Additions:

580633 – Racoons and Rational Numbers



There is a large pile of chestnuts in a forest. Four VERY clever racoons come around the pile, one after the other.

- The first racoon takes half of the chestnuts; then he thinks, well, the rest seems to be too small, and he puts back half of the chestnuts he has taken.
- The second racoon takes half of the chestnuts he finds; then he thinks, well, the rest seems to be too small, and he puts back the third part of the chestnuts he has taken.
- The third racoon takes half of the chestnuts he comes up with; then he thinks, well, the rest seems to be too small, and he puts back the third part of the chestnuts he has taken.
- The fourth racoon again takes half of the chestnuts he finds; then he thinks, well, the rest seems to be too small, and he puts back half of the chestnuts he has taken.

Now the first racoon comes around again and counts the chestnuts that are still there: There are 30 chestnuts left. (As I said, VERY clever racoons.)

- a) How many chestnuts had there been in the beginning?
- b) What if the first racoon in the end counted 32 chestnuts?