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School Math vs. Competition Math

Terese M. O. Nielsen
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January 22nd, 2021

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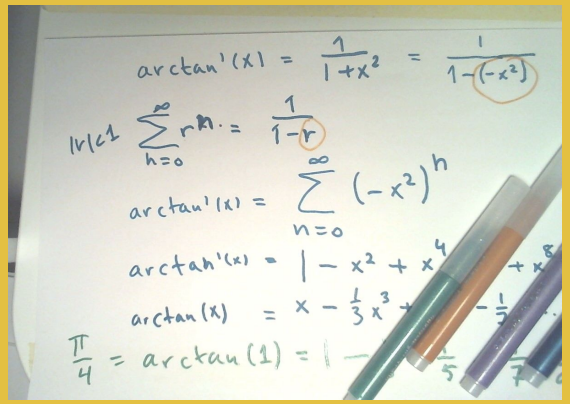
Why is competition math hard?

- even for kids who are normally good at math?

- Me
- My Workplace
- Origins of the question
- Looking at math problems
- The kids' reactions
- What is the relation between competition math and school math?



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Terese M. O. Nielsen

I love explaining stuff

2018-present	Teacher at Science Talenter
2013	Minor in Physics
2003-2016	Teaching at Gymnasiums, Teaching Assistant at Universities
2003	Ph. D, Philosophy of Mathematics
1992	First job as a teaching assistant
1990	Started studying maths and philosophy

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Science Talenter

Organising camps for talented youths

- ages 13-20 with an interest in Science
- picked out by their teachers
- 3 days, eating and sleeping at Sorø (in Covid-19 free times)

Funding:
Mix of government funding, private funds, and payment from the schools.

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Some of the brightest kids don't like competition math *at all*!

They are probably even good at school math...

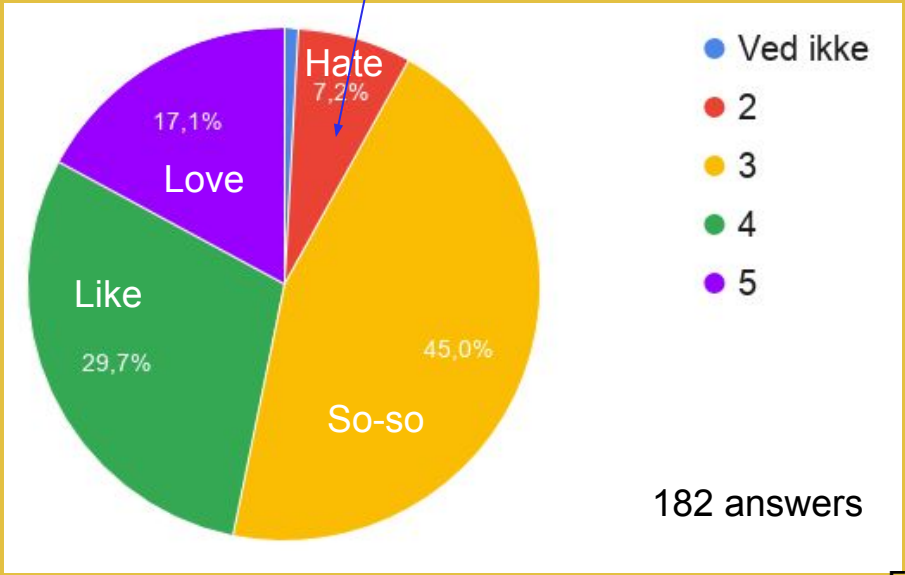
Why is competion math hard?

...even for smart kids

Evaluations from *24 Hours of Science*

- One workshop out of a broad range of STEM subjects
- Math Competition problems
- 8th grade pupils, interested in science
- pointed out by the schools as the most talented

"How would you rate the Georg Mohr Workshop on a 1-5 scale with 5 being best?"



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The Georg Mohr competion

Danish competition, first step in qualification for the Math Olympiad

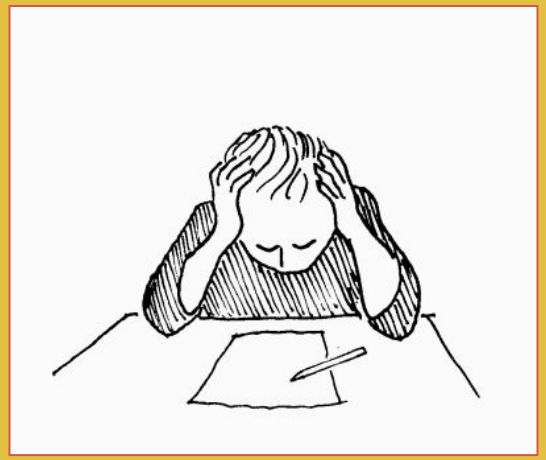
First round in November

- 20 problems, no calculator, multiple choice or an integer answer
- 15.000-20.000 participants, aged 13-20

Second round in January

- 5 problems demanding mathematical argumentation
- 800-1400 participants, almost exclusively Gymnasium

A treasure trove of problems!



https://www.georgmohr.dk/

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Comparing problems

by asking at a Math Camp

- Online
- 24 pupils, aged 14-16
- November 16th-18th 2020
- Participation in the Georg Mohr competition on Nov 17th

Camp om Georg Mohr matematik

På campen vil vi arbejde matematikopgaver, der er nemme at forstå, men svære at løse. Du vil opleve matematik for matematikkens egen skyld. Matematik, der er sjovt, smukt - og svært!
Tirsdag deltager alle i første runde af Georg Mohr konkurrencen.
Se mere på http://www.georgmohr.dk/

Undervisningen finder sted via Google Meet
https://meet.google.com/btp-vyqp-qnw

Program

Mandag d. 16. november

Kl. 8.50-9.00
God morgen. Mulighed for at logge ind og teste teknikken.

Kl. 9.00-11.00
Hvad er det særlige ved matematik? Hvad er det særlige ved Georg Mohr opgaver?
Oplæg og opgaver ved Terese - med små pauser undervejs.

Kl. 11.00-12.00
Pause. Spis frokost og bevæg dig lidt

Kl. 12.00-14.00
Træning. Vi regner Georg Mohr opgaver som forberedelse til konkurrencen

kl. 14.00 Tak for i dag

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Examples

Which subjects are shared?

- Equations
- Geometry and measurement
- Reduction, algebra
- Parentheses and signs
- Probability
- Statistics

Caveat. This is all in a Danish context.
Both school maths and the process of selection for Maths Olympiad may very well be different in other countries.

UNDERVISNINGS MINISTERIET
STYRELSEN FOR UNDERVISNING OG KVALITET

Matematik FP9

Folkeskolens prøver

Prøven uden hjælpemidler

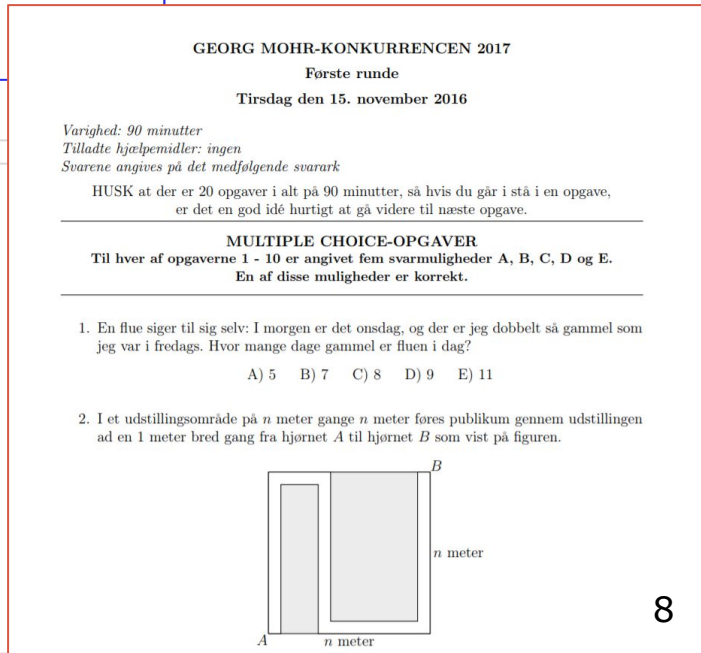
Torsdag den 3. maj 2018
kl. 9.00-10.00

School Test

Exam after 9th grade

GM

Georg Mohr



GEORG MOHR-KONKURRENCEN 2017

Første runde

Tirsdag den 15. november 2016

Varighed: 90 minutter
Tilladte hjælpemidler: ingen
Svarene angives på det medfølgende svarark

HUSK at der er 20 opgaver i alt på 90 minutter, så hvis du går i stå i en opgave, er det en god idé hurtigt at gå videre til næste opgave.

MULTIPLE CHOICE-OPGAVER
Til hver af opgaverne 1 - 10 er angivet fem svarmuligheder A, B, C, D og E. En af disse muligheder er korrekt.

1. En flue siger til sig selv: I morgen er det onsdag, og der er jeg dobbelt så gammel som jeg var i fredags. Hvor mange dage gammel er fluen i dag?

   A) 5 B) 7 C) 8 D) 9 E) 11

2. I et udstillingsområde på n meter gange n meter føres publikum gennem udstillingen ad en 1 meter bred gang fra hjørnet A til hjørnet B som vist på figuren.

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Equations

The School Test problem comes under a heading: “Numbers and algebra”.
The GM is a word problem. No “x=..”

The GM problem needs analysis: what does the age tomorrow and last friday tell you about the age today?

Tal og algebra

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Opgave 7

7.1 $6x + 4 = 28$ $x =$ _____

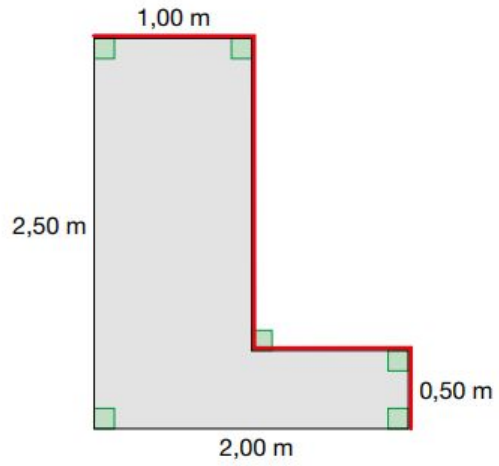
School Test

1. A fly tells itself: Tomorrow is Wednesday. Then I’ll be twice as old as last friday. How many days is the fly of age today?

A) 5 B) 7 C) 8 D) 9 E) 11

Georg Mohr

Geometry and measurement

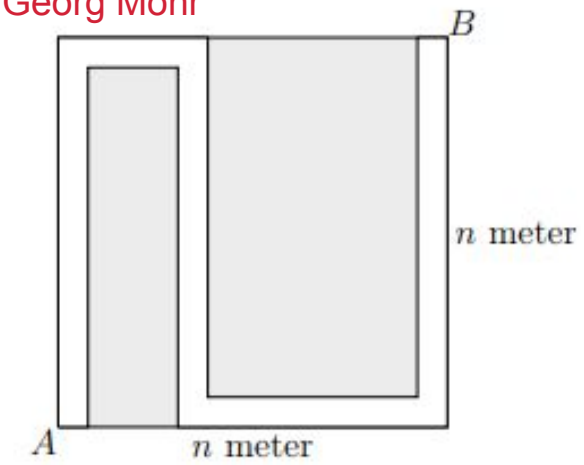


Both problems are dressed up as “practical” problems

Differences
Decimals vs. symbol *n*

Asking for what is seen vs. what is left

Georg Mohr



A table needs a metal edge along the sides marked in red.

School Test

12.1 How many metres of metal edge will der be?
12.2 How many square metres is the table?

In a showroom of *n* times *n* metres, the public is walking through the exhibition on a 1 meter wide path from the corner A to the corner B.

How many m^2 is left for the exhibition, when the area of the path is removed?

A) $n^2 - 3n + 1$ **B)** $n^2 - 4n + 3$ **C)** $n^2 - 4n + 3$
D) $n^2 - 4n$ **E)** $n^2 - 3n - 10$

Reduction, algebra

Differences

School Tests is about a known formula.

GM is about in-equalities: a new subject.

Opgave 9

Arealet af en trekant kan beregnes med formlen $A = \frac{g \cdot h}{2}$.

9.1 Hvilket af de fem udtryk er en korrekt omskrivning af formlen?

Sæt et X.

- ☐ $h = \frac{A}{2 \cdot g}$
- ☐ $h = \frac{g}{2 \cdot A}$
- ☐ $h = \frac{2 \cdot g}{A}$
- ☐ $h = \frac{2 \cdot A}{g}$
- ☐ $h = 2 \cdot A \cdot g$

4. Om tre positive tal x, y og z ved vi at $2x > 3y > 4z$. Hvad kan med sikkerhed sluttes?

A) $x < y < z$ B) $x > 2y > 3z$ C) $3x > 4y > 5z$ D) $3x > 5y > 7z$ E) $x > 3y > 5z$

Of three positive numbers x, y and z is known that $2x > 3y > 4z$. What may we conclude with certainty?

Georg Mohr

The area of a triangle can be calculated as $A = \frac{g \cdot h}{2}$.

Which of the five expressions is correct?

School test

Signs, parentheses

6.2 Hvilket regneudtryk har den mindste værdi?

Sæt et X.

- ☐ $(-3)^2$
- ☐ $0 - (-8)$
- ☐ $-6 + 2^2$
- ☐ $(-4)^2 - 6$
- ☐ $(-2)^3$

Which expression has the lowest value?

School test

Of the number *a* is known that

$$2017a^{2017} + 100 = 110.$$

What is

$$2017(-a)^{2017} + 100\ ?$$

Georg Mohr

Differences

School Tests has exponents 2 and 3.
GM has 2017 as a power.
GM includes *a*.

GM is about parity.

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Examples

Which subjects are NOT shared?

- Arithmetic, only in School Test
- Logic, only in GM
- Game theory, only in GM

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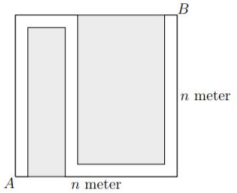


Arithmetic, only in School Test

Opgave 4

4.1 287 + 10013 = _____

4.2 801 – 499 = _____

4.3 102 · 18 = _____

4.4 3648 : 12 = _____

Opgave 5

5.1 11,45 + _____ = 13

5.2 10 · _____ = 5

5.3 $\frac{1}{8}$ + _____ = $\frac{3}{4}$

...is a pre-requisite, not a subject in GM.

School test

Logic, only in GM

Anders, Benjamin, Carla and Dagmar are either always telling the truth or always lying. They say

Anders: *Dagmar lies.*
Benjamin: *Carla lies.*
Dagmar: *One of Benjamin and Carla tells the truth, the other one lies.*

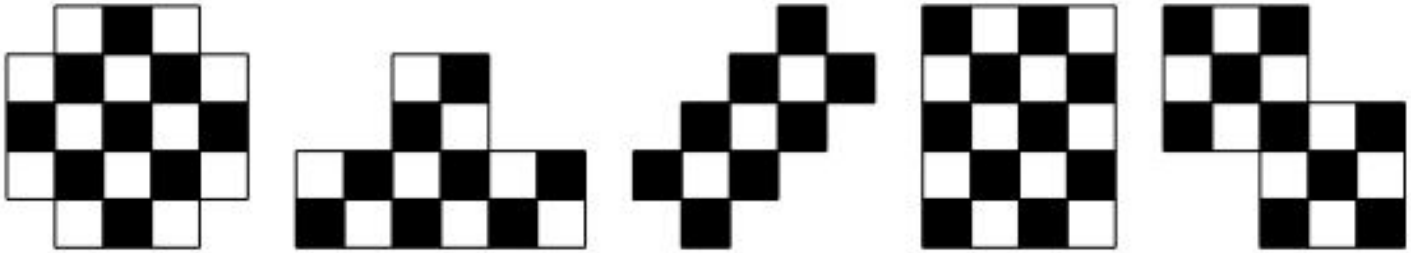
Who do we know for certain always lies?

A) Anders B) Benjamin C) Carla D) Dagmar E) Everybody

Georg Mohr

Game Theory, only in GM

In a single player game, the board has black and white squares. In each turn you choose a row or a column. In this row or column, all black squares will be changed to white and the white ones to black. You win by turning all squares black or all squares white in 4 turns or less. In how many of these boards is it possible to win?



A) 1 B) 2 C) 3 D) 4 E) 5

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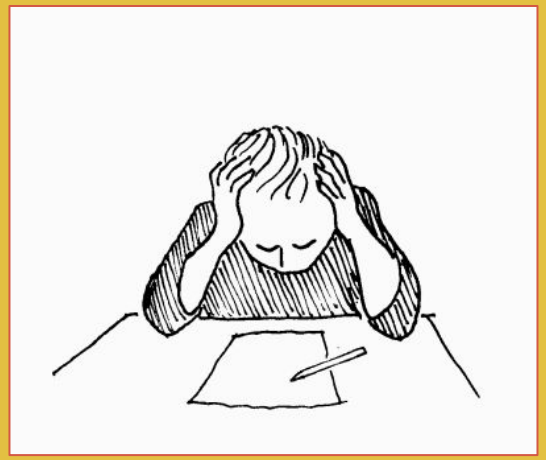
Georg Mohr problems.

How are they different?

Kids reactions:

- lots of text
- using letters like *n* or *a*
- using logical reasoning
- “the answers make sense”
- “you have to get an idea”



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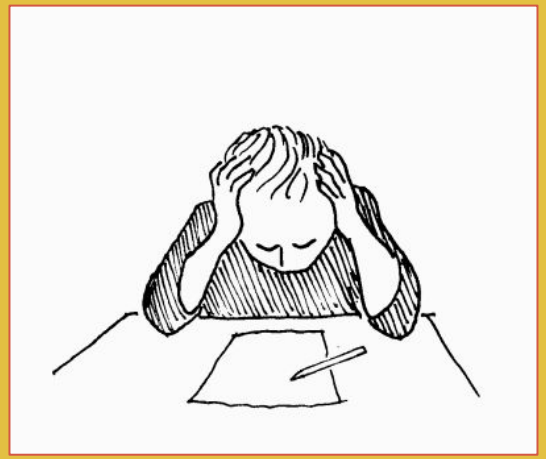
Why is it hard?

Probably a combination of…

- Language
- Complexity
- Ideas

Thank you for your time!
Questions?



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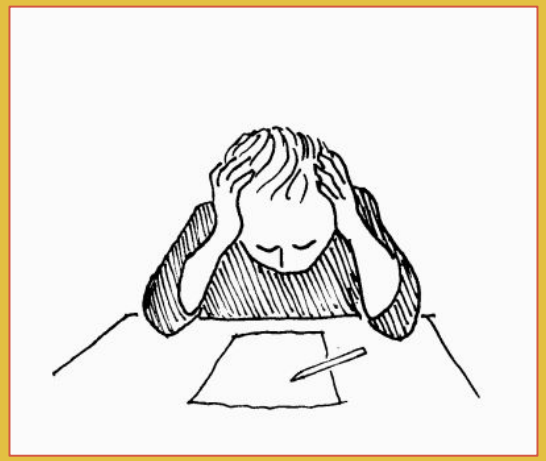
Georg Mohr problems

How are they different?

Teachers' reflections

- Language. Mathematical idioms.
- Complexity. You need to figure out a piece of information not explicitly in the text.
- Complexity in combining more than one subdisciplin: geometry *and* algebra
- No titles. You need to figure out which subdisciplin for yourself.
- Arithmetic, calculation is a prerequisite, not a subject



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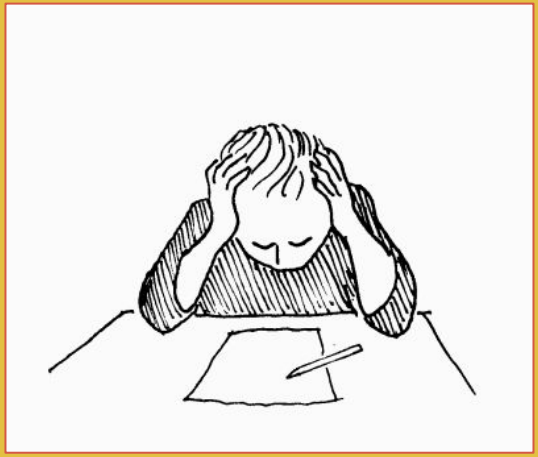
Style vs Content

What is math according to the kids?

- algebra
- geometry
- probability
- statistics
- Georg Mohr problems

An observation
For the rest of the camp, they talked of Georg Mohr problems as a subdiscipline of mathematics, not a style.



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“To promote interest in and increase recruitment to science”

...And it works

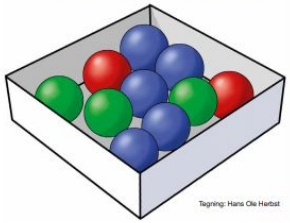
Sylvester, 19 years old

- studying physics and astronomy
- first camp 2015

Probability

Opgave 18

I en æske er der 2 røde, 3 grønne og 5 blå kugler.



Ellen skal trække en tilfældig kugle fra æsken.

18.1 Hvor stor er sandsynligheden for, at Ellen trækker en grøn kugle? _____

18.2 Hvor stor er sandsynligheden for, at Ellen ikke trækker en rød kugle? _____

Når Ellen har trukket en kugle, lægger hun den tilbage i æsken igen.
Derefter trækker hun en kugle igen.

18.3 Hvor stor er sandsynligheden for, at Ellen trækker en blå kugle begge gange? _____

Differences

FP9 has numbers as answers.
GM is asking a question with no answer.

Georg Mohr

Playing heads or tails and writing down A or B.
Which combination is the most probable?

Drawing balls from a box.

School test

5. Georg slår plat eller krone med en mønt. Efter hvert kast skriver han A hvis han fik krone, og B hvis han fik plat. Hver gang tilføjer han det nye bogstav efter det foregående. Efter fire kast har han et ord på fire bogstaver. Hvilket af følgende ord er der størst sandsynlighed for at han har?

A) ABBA B) BBAB C) ABAB D) AAAA E) alle ordene er lige sandsynlige

Statistics

Opgave 19

Herunder er et datasæt, der består af 10 tal.

1, 2, 5, 4, 3, 1, 2, 2, 2, 5.

19.1 Datasættets størsteværdi er _____

19.2 Datasættets median er _____

Tabellen herunder skal udfyldes med tal fra et datasæt, der har middeltallet 4.

10	3	1	5		2	4	5

19.3 Hvilket tal skal der stå i tabellens tomme felt? _____

Asking for mean, max, min, median.

School test

Introducing “running mean”.

Georg Mohr

19. Hver morgen noterer Esben med en rød pen sin vægt i gram i sin lommekalender. Han beregner også hver dag gennemsnittet af dagens vægt og de to forudgående dages vægt og noterer dette tal med blåt. Hver søndag beregner han gennemsnittet af hele ugens røde tal og gennemsnittet af hele ugens blå tal.

Hvis man antager at Esbens vægt aldrig ændrer sig mere end højst 700 gram på et døgn, hvad er så den største forskel angivet i gram der kan være på det røde og det blå ugegennemsnit?

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Subdisciplins

Arithmetic	Algebra	Geometry	Probability	Statistics	Logic	Game Theory
Basic calculation	Manipulation of symbols according to rules	Forms and distances	Quantifying what might happen	Characterising large datasets	What may be deduced from what	Winning and loosing, given precise rules
+ - □	x, y, a, b ...	triangles, circles...	dice, balls in boxes	diagrams, rows of numbers		